

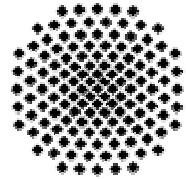
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# The Einstein relation generalized to nonequilibrium

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# Einstein relation

A. Einstein Ann. Phys. **17**, 549 (1905)

**Brownian particle:** (colloidal particle)

diffusion and dissipation have  
the same origin

$$\mu = \frac{1}{k_B T} D$$

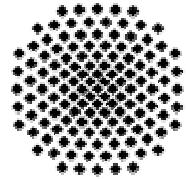
$D$ : diffusion coefficient  
 $\mu$ : mobility

relation generally valid close to equilibrium

**non-equilibrium:**

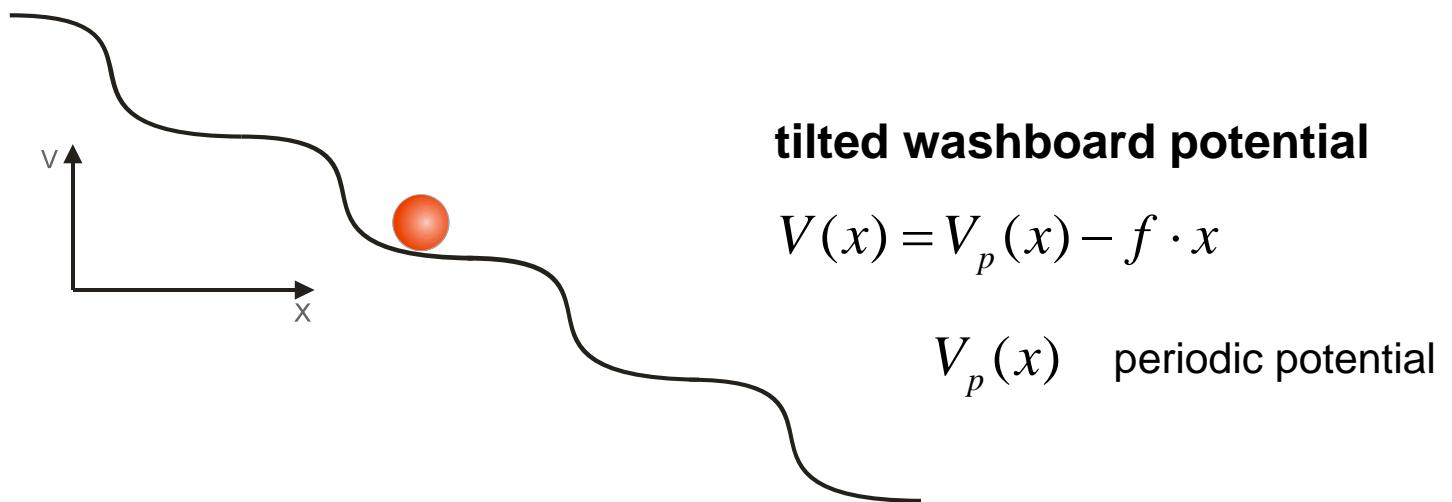
$$\mu \neq \frac{1}{k_B T} D$$

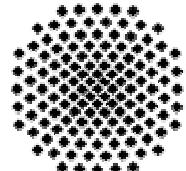
the Einstein relation is violated



## non-equilibrium stationary state NESS:

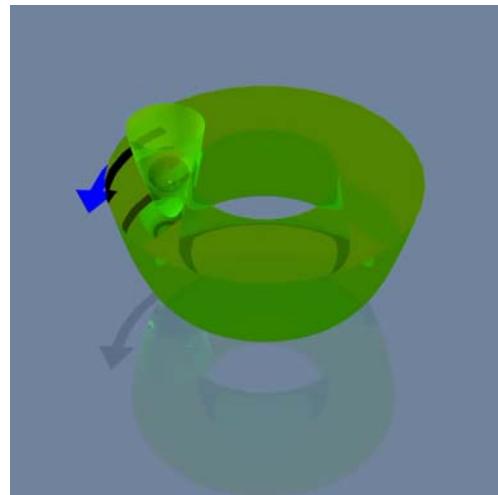
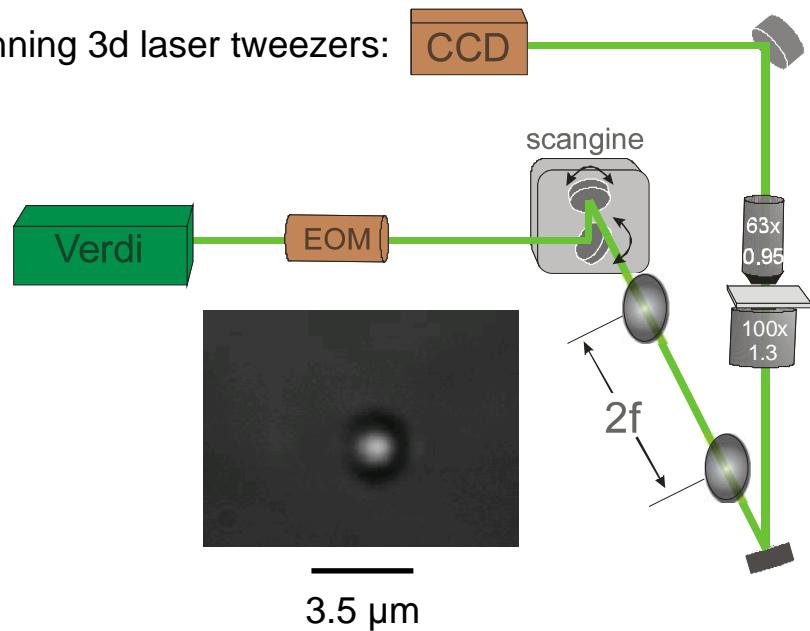
- most simple non-equilibrium situation
- no time dependence (stationary state)
- stationary probability distribution



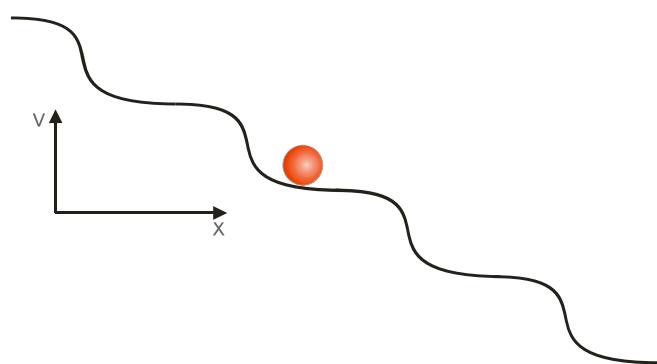
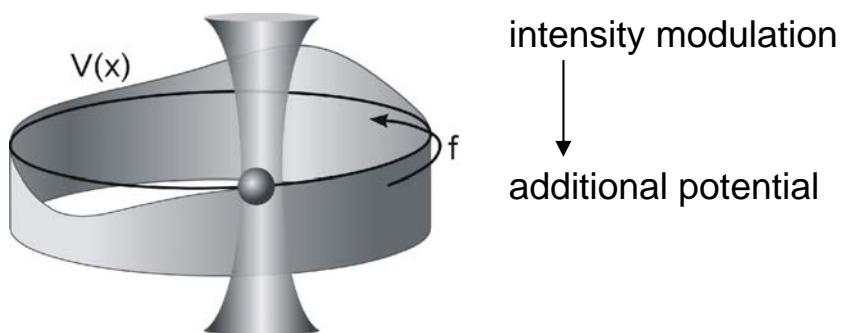


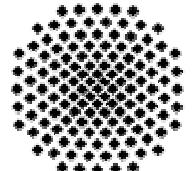
# experimental setup

scanning 3d laser tweezers:



- rotating laser tweezers 570 Hz
- phase-slip regime
- 1d toroidal trap





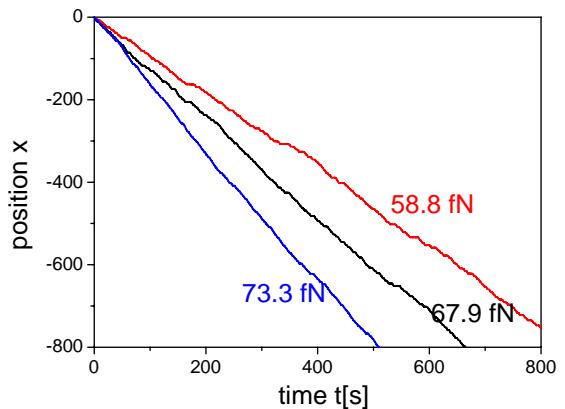
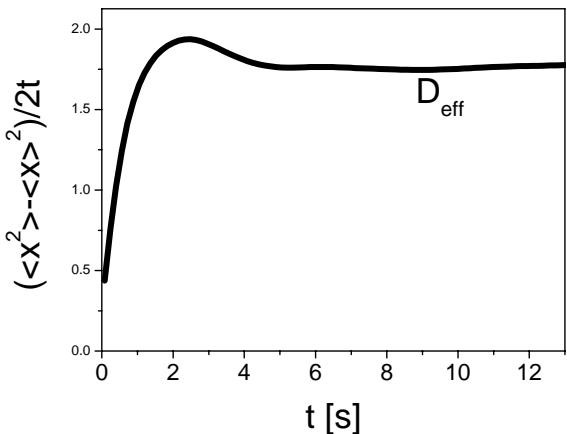
# diffusion and mobility

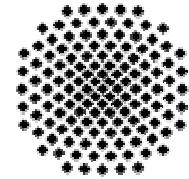
$$D_{\text{eff}} = \lim_{t \rightarrow \infty} \frac{\langle x^2(t) \rangle - \langle x(t) \rangle^2}{2t}$$

$$\mu = \frac{\partial \langle v \rangle}{\partial f} = \frac{\Delta \langle v \rangle}{\Delta f}$$

this definition is independent of  
the drift velocity and the external driving.

measuring drift velocity for different driving forces

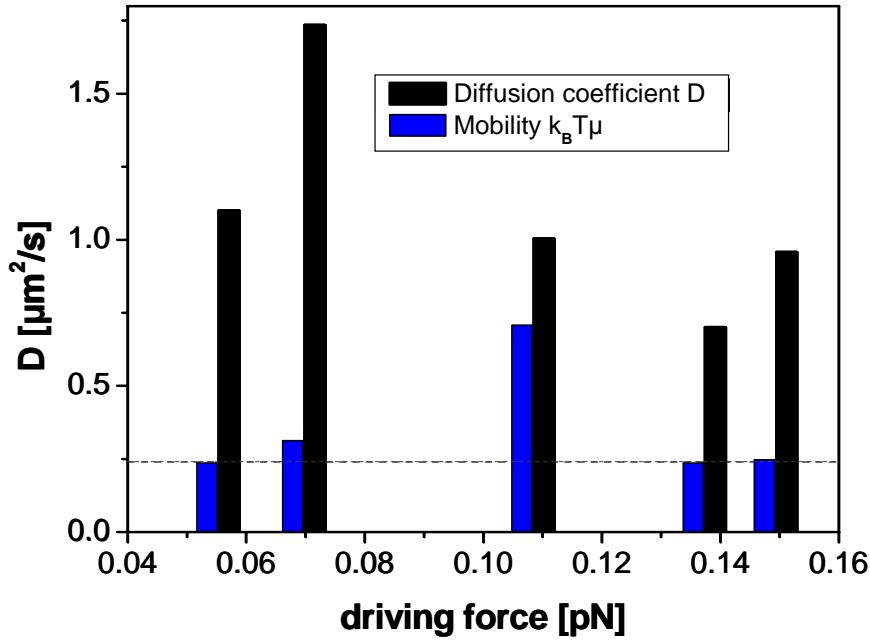




# test of ER

$$\mu \neq \frac{1}{k_B T} D \quad ?$$

Einstein relation is violated



enhancement

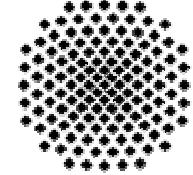
peak

giant diffusion

P. Reimann, PRL **87**, 391 (2001)  
S. Lee, PRL **96**, 10601, (2006)

Can we quantify the violation of ER ?

$$D_0 = 0.24 \mu\text{m}^2 / \text{s}$$

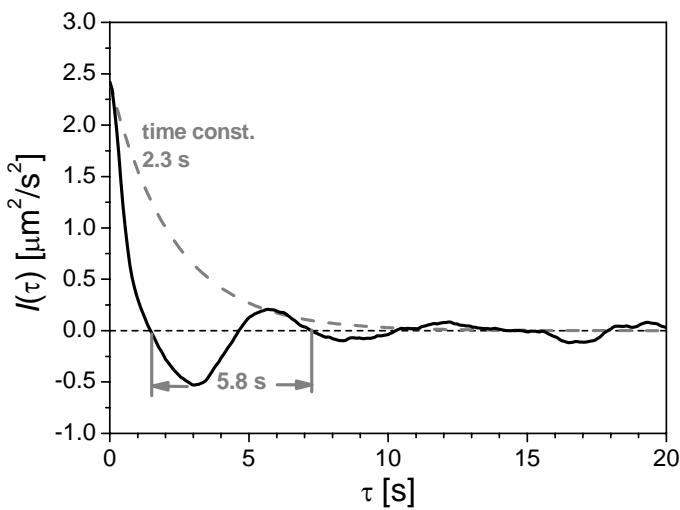
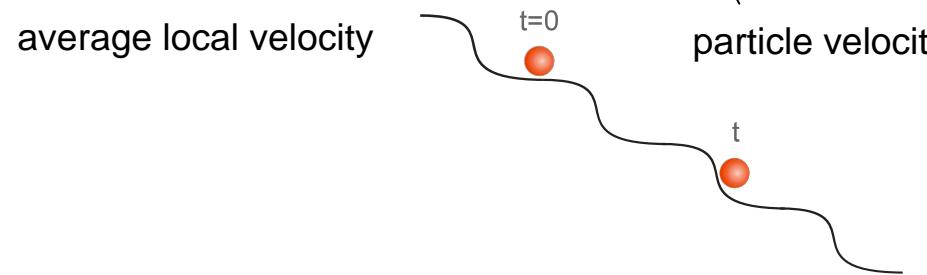


# restoring einstein relation

T. Speck, U. Seifert, Europhys. Lett. 74, 391 (2006)

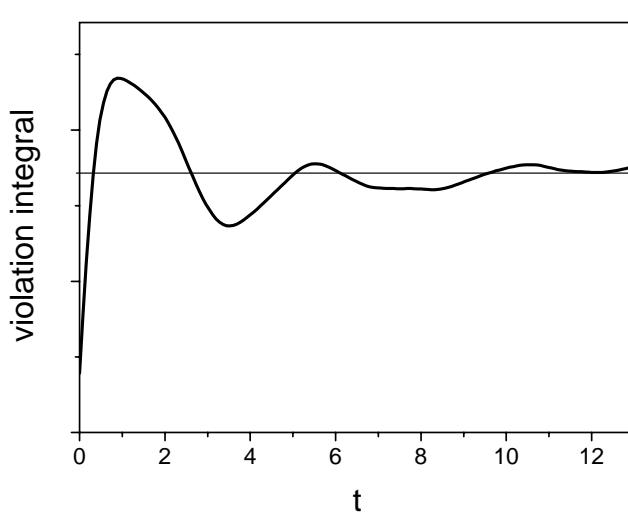
$$D = kT\mu + \int_0^\infty d\tau I(\tau) \longrightarrow \text{violation function} \quad I(t) = \langle v_s(x(0)) \dot{x}(t) \rangle - \langle v \rangle^2$$

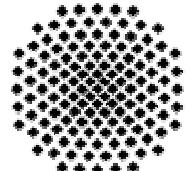
average local velocity      particle velocity



Integration

$$\int_0^t I(\tau) d\tau$$

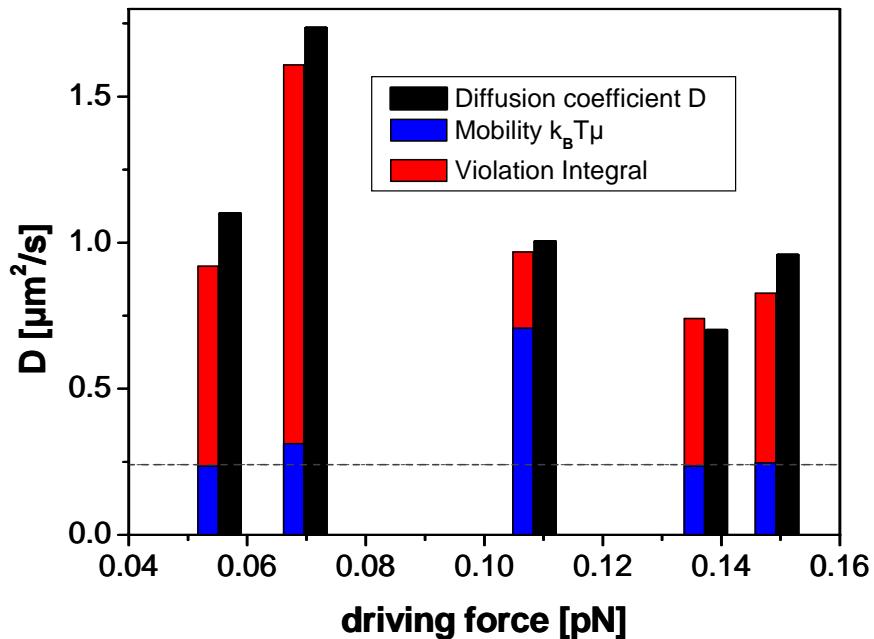




# restoring the ER

$$D_{eff} = kT\mu + \int_0^{\infty} d\tau I(\tau)$$

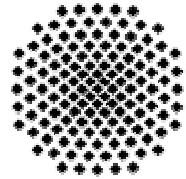
good agreement



violation contributes substantially to D

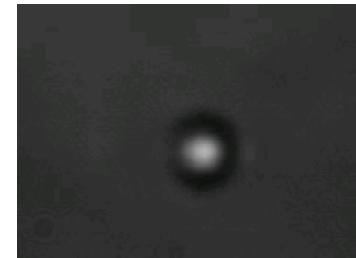
beyond linear response

D and  $\mu$  structures are understood



## summary and outlook

realization of NESS using laser tweezers



test an extended Einstein relation

Langevin approach is suitable to describe colloids in non-equilibrium

general test of fluctuation dissipation theorem

time dependent processes

**Thank you for your attention !!**